Hy[perbolic] par[aboloid]
Matter and geometry in a generative dialogue

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This paper describes our exploration of the Hyperbolic paraboloid surface departing from a computational parametric description to the final materialization both from sheet material and as cast objects. In our research the hyperbolic paraboloid is an instrument to explore the relationships between the virtual, mathematical form and the

Fig.1
Concrete component casted from dynamic formwork.

materialized object in a bi-directional design process. We produced prototypes and developed a dynamic formwork that enables casting of concrete objects with various shapes using one formwork (Fig. 1). The approach is an alternative to the repetitive notion of conventional rigid formwork systems and the formal limitation of flexible fabric formwork. Constraints that originate from fabrication, construction and economy became the design driver in our research.

Hyperbolic paraboloid

A hyperbolic paraboloid is an infinite surface in three dimensions with hyperbolic and parabolic cross-sections. It is often referred to as "saddle" surface because of its anticlastic shape i.e. its curvature in opposite directions. It can be created by reflecting a parabola about its vertex tangent and translate the rotated copy along the original one (Pottmann, Asperl et al. 2007). The parametric equation to describe this type of surface mathematically is:

$$Z=Ax^2-By^2$$

The hyperbolic paraboid surface is a doubly curved and doubly ruled surface; it is doubly ruled because in every point there are two distinct lines that lie on the surface.

In this paper we use the term hypar as coined by Heinrich Engel in 1967 (Demaine, Demaine et al. 1999) to refer to a partial hyperbolic paraboloid shape, which is trimmed from the full infinite surface (Fig. 2). In our computational model the surface curvature became a variable parameter while the length of its perimeter has been kept constant.

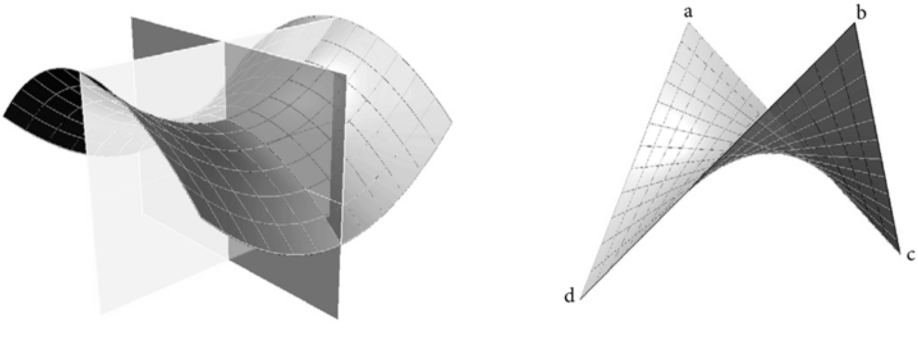

Fig.2

A hyperbolic paraboloid (left), a partial hyperbolic paraboloid shape/hypar (right).

Materializing the hypar

Some of the most obvious ways of materializing the virtual hypar are:

1. Tensing a non stretchable fabric at its four corners by pulling and fixing its two diagonally opposite corners in opposite direction of the other two diagonally opposite corners;
2. Spanning and tensing a string network along a frame whose shape represents the outline of hypar surface;
3. Distributing wood rods along a frame whose shape represents the outline of hypar surface (Fig. 3).

On our way towards materialization we sought to correlate the double curvature of the hypar with constraints originating from producing them from sheet materials. Negotiating these opposing points of departure became the challenge of our embedded-rationale (Whitehead, Peters et al. 2004; Peters 2008) design exploration. In contrary to post-rationalization where the geometric and constructional rationalization takes place after the design exploration, embedded-rational exploration is when “the geometric systems and constructional logic is established as an integrated part of the design process” (Peters 2008). In such design processes, physical and digital models become more than a representation and would simulate certain behaviors as the result of the embedded geometric and construction logic. In such processes choice of material for physical modeling and the way of creating the models have significant impact on the product being produced.

Folding the hypar

The oscillation between the doubly curved surface in the computational model and paper as a sheet material was facilitated through folding: A flat square paper can be transformed into a hypar through a sequence of folding procedures (Fig. 4). As folding progresses from sides of the paper to its center, the paper buckles and takes the shape of hypar.

In contrast to fabric, paper always maintains its in-plane stiffness and does not require a cumbersome framework. Folding paper became an intuitive, controllable, generative and fast way to explore various shapes. Similar to Kurt Kranz we searched for “rules, principle and procedures or processes that generate changes by artistic means but lead to consistent processes that can be controlled intuitively” (Hiller 2011).

Inspired by Josef Albers paper folding exercises (Horowitz & Danilowitz 2006), our Hyper surface was materialized in paper and became a vehicle for the exploration of serially differentiated components. The folding procedure allowed us to materialize the parametrically computed hypar and maintain its geometric flexibility, similar to changing the distance of the parabola end points in the digital model. We pushed and pulled the two opposite corners of a paper hypar to generate various figures (Fig.5).

Fig. 3
Materializing the hypar with fabric, string network and wood rods.

Fig. 4
1. Start with folding the two diagonal creases on a square paper. 2. Fold a two-directional corrugated accordion pattern and unfold again to receive concentric squares. 3. Fold a ridge-valley pattern staring from the corner this time. 4. After folding all four corners the planar sheet automatically curves and takes the shape of hypar.

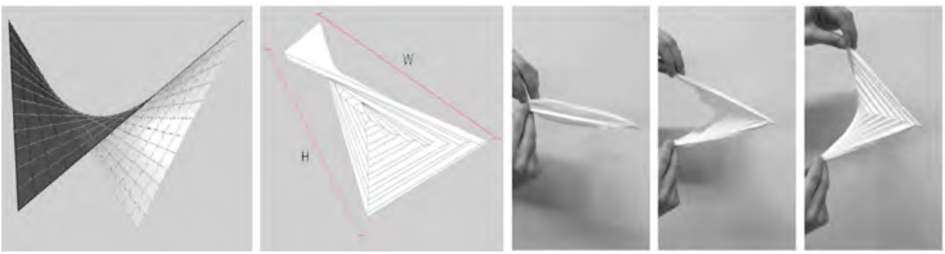

Fig. 5
Differentiation of computed and paper hypar.

Generating and casting shapes

To study and explore the variant figures generated by the folded paper model, we froze every figure through casting, then produced a whole series of components with equal perimeter length but different curvature. The folded paper model, used as a vehicle for parametric exploration, turned into a mould for gypsum casting. This collection of components represented a whole range of possible shapes originating from

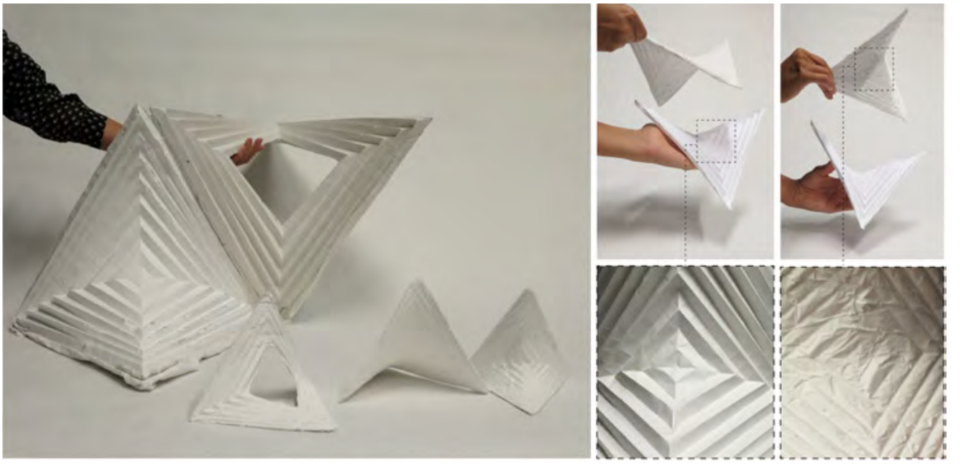

Fig. 6

Archive of casted hypar components (left), quick casting using paper as a formwork (up-right) and the traces of fold on the casted gypsum (down-right).

one paper formwork. The folds, necessary to transform paper into a hypar, left their traces on the component (Fig. 6).

Simulating the folding of the paper

These folds, developed to overcome the conflict between the three-dimensionality of the hypar and the planarity of the paper, became a design feature. Subsequently, we fed them back into our original parametric model, which was previously driven by the parabola and hyperbola curves only. The next model iteration incorporated the concentric square creases and the ridge and valley folds of the paper model (Fig.7). The major effort here was to design an algorithm that simulates the folding of the paper. The new computational model now folds from the inside out: A change in the central fold drives every fold outwards (Fig. 7). A twisted patch surface represents the twisted rectangular stripe of the paper hypar.

Folding plastic sheet and casting gypsum

The second iteration of the computational hypar was unfolded and the pattern was engraved into a plastic sheet material that could better withstand the liquid gypsum. Through folding the squared plastic sheet, the casting bed of formwork for gypsum casting was produced (Fig. 8).

While folded plastic performed well for casting gypsum, it could not withstand the hydrostatic pressure of concrete. On the other hand its stiffness provided far less flexibil-

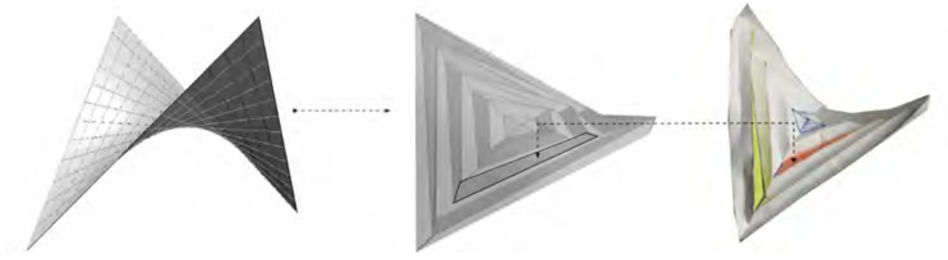

Fig. 7
The first parametric computational model driven by parabola and hyperbola curves (left), the second iteration of computational model driven by central fold (middle) and the twisted surface between ridge and valley fold of the paper model.

Fig. 8
The casting bed made from folded paper (left) and the gypsum casting (right).

ity than the paper hypar during the exploration of different shapes, which then limited the geometric range of cast objects.

Computational discretization of the fold

The previous experiments clearly revealed the central contradiction and challenge of our research: The oscillation between the aim for flexibility during formal exploration versus the need for rigidity in the formwork. We needed a strong material that prevents the formwork from bulging under the load of concrete and at the same time a kinetic movement in the formwork to produce components of variant curvature economically. The increased demands on material strength caused by concrete led to a reconfiguration of the folded hypar. Instead of paper or plastic we chose 3mm MDF for the materialization of the hypar concrete formwork to tackle the increased hydrostatic pressure of concrete.

In contrary to paper and plastic formwork that twist between each ridge and valley, the poor compression and shear properties of MDF do not allow it to twist. To overcome this problem the third computational model was developed to rationalize the fold pattern to meet our material capability. We replaced the twist by another diagonal

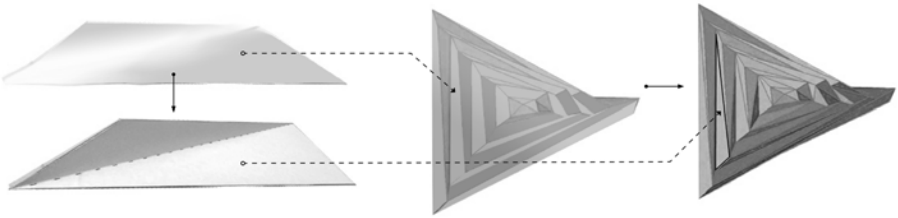

Fig. 9
Rationalising the twisted surface by translating the twisted surface to two triangles.

Fig. 10
The bed of formwork as MDF hypar, advance dynamic formwork, and casted concrete hypar.

fold that transform a twisted surface into two triangles, enabling the production of the Hyper formwork from non-twistable sheet materials (here: 3mm MDF). This third computational model reflected the rationalization and simulated the additional folding (Fig. 9).

Dynamic formwork and casting the hypar with concrete

The hypar formwork could now be CNC-cut from non-twistable materials (here: 3mm MDF) with textile-reinforced tape acting as hinge in between (Fig. 10). We placed two of these folded hypars with a distance of 20mm into a box and cast in-between. Two elevated and opposing faces of the box were cut into triangles that could move along this diagonal cutting seam causing the hypar formwork to change its shape. Thus the formwork became dynamic and allowed for the casting of differently shaped hypar concrete objects. The triangulated folds of the formwork left their trace on both sides of the cast object.

From folding to dynamic formwork

The mechanical system of dynamic formwork originated from the initial material system developed for folding paper. The shift from gypsum to concrete and an increase

Fig. 11
The shift from paper to MDF material led to shift from folding to hinging mechanism. At the far left is the concrete component casted from the MDF hypar.

in scale required a shift in formwork material. Consequently the kinetic mechanism shifts from folding and twisting in paper and plastic, to hinging in MDF and textile reinforced tape (Fig. 11). Both approaches became systems of dynamic formwork, which challenge the repetitive notion of conventional rigid formwork systems and the formal limitation of flexible fabric formwork.

The choice of paper and folding in the exploration of hypar was adequate to our design intention and knowledge of envisioning certain characters for dynamic formwork. Dynamic formwork systems as we are investigating them in our research rely on various mechanisms, material capacities and kinematics that allow them to change shape. Hence, one formwork can be used to cast differently shaped concrete objects: Through folding and hinging we were able to produce various hypar shapes from one formwork system. These particular character of dynamic formwork distinguishes it from fabric formwork. Fabric formwork is one of very few examples that relies on the self-organisation of concrete within a system of external forces, material capacities of fabric and the viscosity of concrete. Form emerges within a process of becoming. Fabric formwork is considered as flexible formwork that relies on textile under tension due to the hydrostatic pressure of concrete. A tensioned membrane deflects into a catenary curve or shell-like surface constrained by a fixed boundary (Manelius 2012). Here flexibility leads to forms of minimal energy consumption with the predefined boundary conditions. However the range of shapes is limited by the single-objective optimisation. Through the use of dynamic formwork we seek to enlarge the possibilities for differentiation.

Integrating form, formwork and process

The integration of formwork design into the architectural design process is driven by the idea of creating and understanding material system. A material system is a structure with spatial, loadbearing, energy storing or energy conducting properties, which are derived from the particular material it is made of and from the process it is produced with. Computational tools and techniques, furthermore, allow to notate and instrumentalise the intricate interactions between form, material, structure and environment within the architectural design process.

Simulating material systems within digital generative models utilises computation beyond formal and geometrical design schemes. The notion of the model shifts from representation of objects towards the abstraction of a process and the prediction of behaviour. Simulation integrated into an iterative and circular manner allows feedback to inform the design process (Hensel & Menges 2006). Here we started simulating the formwork folding of planar faces around linear hinges within a parametric model. The digital process did not only provide the necessary geometric information for manufacturing the formwork but also generated the various concrete shapes. Thus the computational model provided both, a tool for design exploration of concrete elements and a system to extract fabrication data and constraints of the formwork system.

Assemble the showcase of hypar

Finally all hypar components came together into an installation as a showcase (Fig. 12). The showcase of the materialized hypar invites visitors to reflect on the role of material and material system throughout the design processes. The components were assembled on a Hyperboloid wood structure that was developed in parallel with the hypar exploration. Components were assembled in accordance to their weight from top to bottom in order of the paper, gypsum and concrete components.

Conclusion

The process from a mathematical description to the materialization of hypars is comprised of iterative feedback loops and a shift between digital and physical exploration. Insights gained on the way from design to production feed back into the design and

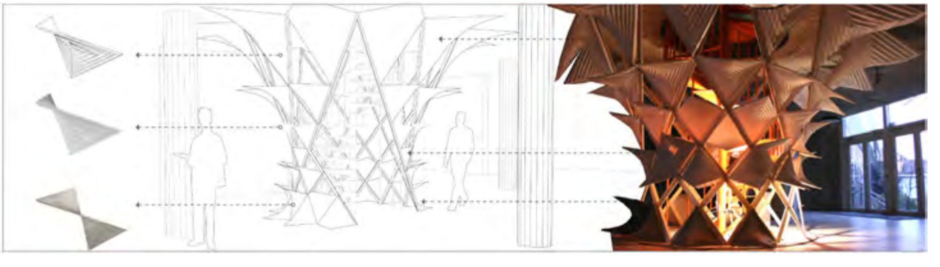

Fig. 12
The installation as a showcase of hypars.

inform the process and design intention. As designers we reflect on the making and through the making. Every produced model, of physical or digital nature, was a step stone for developing the next model in the process and refinement of the design intention.

Throughout this process we consciously sought for performative and aesthetic features emerging within the representation, the simulation and the making. Folding became the vehicle to transfer sheet material into double curved hypars. The ornamental qualities of folding, fed back into the computational model, became operational again as hinges in a dynamic formwork system. Technical necessities and constraints become ornaments and design features. Folding is the starting point for a material efficient dynamic formwork enabling casting different shapes from one system.

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